Student's Learning Obstacles on Mathematical Understanding of a Function: A Case Study in Indonesia Higher Education

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Abstract **- This study aims to investigate conceptual understanding based on the cognitive process and to describe the learning obstacles faced by students to digest the notion of definition of a function. For this purpose, a case study is conducted by involving three undergraduate students of mathematics education program as the subjects. One is a second semester student (M1), and the other two are a sixth semester student (M2) and eighth semester student (M3), respectively. The data are collected using observation, tests, and interviews. We found that M1's process level of cognitive is in remembering. M1 experienced didactical, cognitive, and epistemological obstacles; M2's process level of cognition is in remembering and understanding, which reveals cognitive and epistemological obstacles; M3's process level of cognition is in remembering, which shows cognitive and epistemological obstacles. The results are definitely important to improve the process of teaching and learning at universities. This result is used as an evaluation material for students to be at the Analysis stage in terms of Bloom's taxonomy level. Thus, it will serve as a material for the lecturers to deliver the concept of function emphasizing on formal definition and representation.**

Keywords **- conceptual understanding, cognitive process, learning obstacles, definition of a function, Bloom's taxonomy level.**

DOI: 10.18421/TEM84-44
https://dx.doi.org/10.18421/TEM84-44

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Received: 11 May 2019.
Revised: 05 October 2019.
Accepted: 19 October 2019.
Published: 30 November 2019.

1. Introduction

Definitions have an important and basic role in mathematics and mathematics education [1], [2], and it was later highlighted by mathematicians and mathematical educators that the mathematical definition is different from the "ordinary" definition. In mathematics, the definition can build a mathematical concepts which was the most basic material of mathematics and has become an important topic in research for several years [3], [4]. Therefore, the definition is the basic thing in learning mathematics to build conceptual understanding of a mathematical concept. It is thus in learning the concept of function because this concept is constructed based on the definition of a function.

The concept of a function is a basic concept in mathematics. The function is one of the most important topics in the curriculum and internationally considered as the main theme in the mathematics curriculum and related to other subjects such as physics [5], engineering [6], [7], and astronomy [8]. One characteristic of the concept of function is that it can be represented in various ways (e.g., tables, graphs, symbolic equations, and verbal) and an important aspect of understanding the concept of function is the ability to use various representations and transfer them from one form to another [9]. The concept of a function was perceived as a springboard for deeper and broader mathematical content such as limits, Fourier series, topology, and metric spaces at the university level [10].

Understanding the concept of a function will help the students to solve the problems in mathematics. It is an indicator that the undergraduate students are in learning development. According to [11], understanding the process of knowledge change is a major goal in the development of learning and education. More specifically, [12] have remarked that mathematical understanding can be seen as a process to achieve an understanding and as a result of understanding. So, it is essential for the students to be able to relate all the procedures or facts of mathematical concepts that can create mathematical ideas. Therefore, as remarked by [13], mathematical ideas facts will be more easily understood when these items are considered as part of a network of ideas. Mathematical ideas, procedures, and facts will be comprehensively understood if they are associated with the existing network. For example, networks of a formal definition of a function are sets, ordered pairs, Cartesian product, and relation. Understanding this network will help students to get a conceptual understanding of the formal definition of a function.

In various countries, conceptual understanding is essential in mathematics education; it should be put in curriculum [14]. Overview of conceptual understanding as an understanding of mathematical concepts, operations, and relations is required to help students. Through conceptual understanding, students will understand not only what must be done but also explains why it must be done [15]. Conceptual understanding can be done by building new knowledge based on prior knowledge. The level of student understanding of the concept of a formal definition of a function can be expressed based on prior knowledge of the initial concepts of a function. The following is an understanding of the undergraduate students of Madura University towards the formal definition of a function. Here is an example of how student defines a function in the Indonesian language: *Memasangkan suatu himpunan dengan tepat satu ke anggota lain*. This means: "A function is pairing a set exactly one element with another". This shows that the conceptual understanding of undergraduate students is still based on the understanding in secondary schools and does not yet understand the formal definition of functions which include prior knowledge, namely set, Cartesian product, relation, and initial definition of a function. Whereas conceptual understanding of undergraduate students should be at the level of formal thinking.

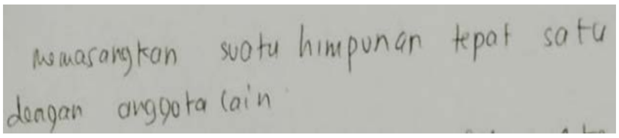

Figure 1. Formal definition of a function undergraduate student

Many studies show that many students have difficulties in understanding a formal definition of function even though they should have been at the level of formal thinking. [16] state that the difficulties are commonly experienced by senior high school and university students. Difficulties faced by these students will create some obstacles in learning how to understand the definition of the function. Obstacles play an important role in learning because they force students to modify and adjust some aspect of their thinking to resolve this contradiction [17]. Obstacles are divided into 4 parts, namely: cognitive obstacles, genetic and psychological obstacles, didactical obstacles, and epistemological obstacles [18]. Cognitive obstacle occurs because of difficulties in the learning process. Genetic and psychological obstacle occurs because of students' self-development. Didactical obstacle occurs because of the nature or method of teaching from a teacher. Epistemological obstacle occurs because of the nature of the mathematical concept itself. The epistemological obstacle in this research occurs when students have a misunderstanding to define and to perform formal definition of a function. Students will be prospective teachers in the future, they must master the mathematical concepts well, especially the concept of functions that is related to many other topics. Based on the level of understanding of Bloom's Taxonomy, students are able to build the meaning of learning messages through oral, written and graphical communication [19].

This research question is how to investigate and to describe the conceptual understanding and the obstacles experienced by mathematics students of Madura University in formally defining that notion.

Due to the important role of the notion of function in mathematics, the purpose of this research is to investigate and to describe in detail the conceptual understanding and the obstacles experienced by mathematics students of Madura University in formally defining that notion.

2. Literature Review

Conceptual Understanding

The two important types of knowledge that a person owns are conceptual understanding and procedural skills. Conceptual understanding is the main objective of mathematics education in the mathematics curriculum in various countries [14]. Procedural skills are the ability to implement procedures accurately, efficiently and flexibly; to use the procedure in various problems and contexts; to build or modify from one procedure to another; and to recognize when a strategy or procedure is more appropriate to implement than others [20].

Conceptual understanding is an understanding that is not only about what must be done, but also explains why it is done [15]. Conceptual understanding can be done by building new knowledge based on prior knowledge that includes the concepts, operations and relationships. Finding out the level of students' understanding of a function definition can be done by expressing the function definition with its own language and giving a description of examples and non-examples.

Definition of a Function

The function is an important topic in mathematics and mathematics curriculum. The function is a basic concept and support in mathematics itself or between mathematics and the real world. The function is the basis for calculus learning that must be studied continuously. So, to be able to truly understand calculus, students must understand the concept of function well. Previous research has shown that mathematics students' understanding of functions develops over a long period of time and many mathematicians are underdeveloped in understanding the concept of a function [21].

Understanding of function was not easy for students, although functions were introduced from the secondary school level. The concept of function is one of the mathematical concepts which are first introduced at the second grade of secondary education and has a central role at the curriculum for the following years [4]. The understanding of functions does not appear to be easy, while students face many obstacles trying to understand the specific concept [9].

Functions can be represented in various ways depending on one's learning level. If a student reaches the college student stage, various representations have developed well. Whereas, according to [22], it represents through diagrams, tables, and verbal descriptions. Functions have different faces and making students perceive them as faces of the same mathematical concept is a pedagogical and research challenge [23]. Function includes the ability to adjust one representation to another, the flexibility to use appropriate representations in problem solving, the ability to use one representation can be used in other fields [24]. For students who have understood the concept of function, they can find relationships between various representations, are able to choose the most appropriate representation in solving problems, and can transfer between representations with relative ease [25].

Function definitions have been introduced from secondary schools to universities. Along with the development of mathematics, it is clear that the definition of "function" which has been understood so far is more general. For this reason, it was revised to find out the definition of the function itself and the value of the function. The revised definition is as follows.

> A function f from a set A into a set B is a rule of correspondence that assigns to each element x in A a uniquely determined element $f(x)$ in B.

The definition still uses the representation of words and is simple, not yet connected with formal mathematics. It also has weaknesses, as revealed by [26] that there is difficulty in interpreting the expression "correspondence rules". To clarify this, then the function definition is connected with the set. The following is formal definition of a function according to Bartle & Sherbet (2011).

> Let A and B be sets. Then a function from A to B is a set f of ordered pairs in $A \times B$ such that for each $a \in A$, there exists a unique $b \in B$ with $(a, b) \in A \times B \in f$. (In other words, if $(a, b) \in f$ and $(a, b') \in f$, then $b = b'$).

The concept of function can be defined formally and symbolically, almost without the use of words. The concept of function admits a variety of representations, while several representations of the concept offer information about particular aspects of the concept without being able to describe it completely [27].

Conceptual Understanding of Formal Definition of a Function

Conceptual understanding of function definition refers to indicators of understanding the definition of functions. An indicator of understanding the definition of functions is as follows: (1) Students' ideas about what the function definition is; (2) Its ability to present functions in different forms; (3) Solving function problems from one representation to another representation [28]. Those indicators of understanding the definition of functions are: (1) Defining the concept of functions and making examples of functions; (2) Complete the task by asking students to recognize and interpret the concept of functions presented in the form of different representation; (3) Solve function problems [4].

Whereas [29] states that the function definition indicators are as follows: (1) Formalizing the mathematical language and dialect he writes; (2) Understanding syntactically correct definitions and meaningful statements represented by mathematical language; (3) Obtaining various examples, starting from small objects to large objects; (4) One way of gaining knowledge of how definitions are used in the theorem and related to other topics. Indicators of understanding the formal definition of functions are: (1) defining functions formally based on their own language and according to the ideas which are based on prior knowledge, (2) presenting functions through examples and non-examples, (3) checking its conceptual understanding by explaining the definition relationship which is made by example and non - example according to the formal definition of the function. Describing in details what has been learned by providing the examples is an indicator of the students' understanding of the concept.

Learning Obstacles

Obstacle is the knowledge that is useful in solving certain types of problems, but if it is applied to new problems or contexts, that knowledge is insufficient or contradictory [18], [30]. While [31] states that obstacle is something that hinders student in learning. Difficulties faced by students can cause difficulty in learning and understanding the definition of functions. The obstacle in learning the definition of a function obstructs or prevent students from understanding a function definition.

These obstacles can spur student learning to understand the full function definition. This obstacle consists of four parts, namely didactical obstacle, epistemological obstacle, cognitive obstacle, psychological obstacle. Didactical obstacle occurs because of the nature of teaching and the teacher, epistemological obstacle occurs because of the nature of the mathematical concept itself, cognitive obstacle occurs when students experience difficulties in the learning process. Genetic and psychological obstacle occur as a result of students' personal development [18].

3. Method Design

This study is to investigate and describe in details the conceptual understanding and obstacles experienced by mathematics students of Madura University when they define a function. This study is a case study approach; it is to describe the characteristics of students' community and to intensively analyze various phenomena that exist. More specifically, it is to describe the obstacles that the students have faced in understanding the formal definition of a function.

Materials

Subjects were given questions to define a function based on their understanding by using their own language (their languages) and provided examples and non-examples. The tests are to focus on (a) the students' ideas about the formal definition of a function, (b) their ability to present functions in a different form from the examples and non-examples, and (c) their conceptual understanding by explaining the definition relationships made with examples and non-examples. The tests are also given to the students of the senior high school to understand and compare their understanding between the students of Madura University and students of senior high school.

Participants

In our case study, a second semester student (M1), a sixth semester student (M2), and an eighth semester student (M3) of Madura University are the subjects of this research. M1 had taken calculus and introductory mathematics, and M2 had taken (had joined) calculus, introductory mathematics, and real analysis while M3 had taken all courses and should have been in the formal stage of understanding. The subjects were chosen based on learning obstacles, communicative ability, and collaborative ability.

Procedures

The researcher observes intensively the teaching method of teachers and the learning process to get learning obstacles of students in formal defining of a function. The tests provided to the students are (a) formally defining function using the student's own language and in accordance with the ideas already owned based on prior knowledge, (b) presenting functions through examples and non-examples, (c) checking the conceptual understanding by explaining the relationships of definitions made with examples and non-examples, and (d) representing in mathematical language (sets, ordered pairs, Cartesian product, relation, example and non-example of a function).

Data Collection Methods

The subjects were observed, tested, and interviewed. Observations were performed by looking at the lecturing process in the classroom, and some tests were given to identify the level of students' conceptual understanding. Meanwhile, the interview was conducted to check the progress of the conceptual understanding has been obtained during senior high school compared to the understanding as well as the types of obstacles experienced by students.

Data Analysis

In analyzing the data, some steps the research uses are (1) identifying the data, (2) verifying the data of students' understanding in formal defining of a function, (3) simplifying the data of students' understanding in formal defining of a function, (4) summarizing or making conclusion of students' learning obstacles and conceptual understanding in formal definition of a function.

4. Findings and Results

From this study, we found that (a) The students do not understand the networks of a formal definition of a function which involves sets, ordered pairs, Cartesian product, and relation; (b) In learning formal definition of a function, the students have faced many obstacles such as described in the second column in Table 1.

Table 1. Learning obstacles of mathematics students at Madura University

Subject	Learning Obstacles	Cognitive Level
M1	Didactical, cognitive, and epistemological	Remembering
M2	Cognitive and epistemological	Remembering and understanding
M3	Cognitive and epistemological	Remembering

(c) M1 have all three kinds of obstacles, namely, didactical, cognitive, and epistemological obstacles. But, M2 and M3 have only the last two obstacles. Didactical obstacles happen to M1 since the lecturer of Calculus and that of Introduction of Mathematics do not give any information of formal definition of a function. They consider the students have the ability to define. Instead, they ask the students to incorporate the definition in routine tasks. As a consequence, the students have difficulties in solving non-routine problems; (d) Cognitive obstacles appear in M1, M2, and M3. This really makes sense since they define a function based on their understanding obtained in senior high school. What they have done is only recalling their prior knowledge without inviting any information of formal definition of a function such as the way to associate between set and ordered pairs, Cartesian product, and relation; (e) Epistemological obstacles are also present in M1, M2, and M3. They have difficulties in understanding the mathematical symbols. Consequently, they have no ability to relate those symbols to define a function. As a result, they are not able to write symbolically such a function.

These obstacles affect students' cognitive level. In fact, the cognitive level of M1, M2, and M3 refers to remembering level. In particular, the level of M2 does not only refer to the remembering level but also to the understanding level. At the remembering level, the definition of a function is represented according to students' understanding from senior high schools. This shows that they have difficulties in bridging their prior knowledge to their new knowledge. Finally, at the understanding level, students are able to represent visually a function in terms of ordered pairs, and algebraic form. These findings will be discussed in more details in the next section.

5. Discussion

Three kinds of obstacles, namely, didactical, cognitive, and epistemological obstacles are revealed among M1, M2, and M3 students of mathematics at the Madura University. If M1 experience all three obstacles, M2 and M3 deal only with the last two of them. To obtain a deeper understanding of students' obstacles, we conducted an interview. Here are the results.

M1's Obstacle

For M1, a function is a relation that maps a set into another set provided that all members of the set of origin (domain) must have exactly one member in another area (co-domain). This means that M1 defines a function in the same way as the definition they learned in senior high school. This is very surprising because M1 has already learned basic calculus in the first semester and introduction to mathematics in the second semester.

Students' understanding is influenced by their experience during their study at senior high school. As we all know, the concept of set, ordered pair, Cartesian product, and relation are not covered in high school. Therefore, it makes sense if they do not understand how to relate these concepts to the formal definition of a function. The students' understanding is still not complete yet, since they base their knowledge on what they received in senior high school. The definition of a function in senior high school is a special relation pairing each member of a set A exactly to one member in the set B.

To confirm M1's understanding, we conducted some tests to senior high school students. The results show that M1's understanding and senior high school students' are the same. It proves that they have the same level of understanding. They emphasize that a function is a relation from domain to co-domain that pairs each member of the domain with exactly one of the elements of the co-domain. Thus, in terms of cognitive obstacles, M1's understanding is at the level of senior high school understanding. The students do not have any development in the formal definition of a function; they are static. M1s were not able to associate new knowledge with prior knowledge.

This phenomenon is clear because the nature of the cognitive obstacle is closely related to solving any kind of mathematical task [32], particularly in problematic types of tasks. To overcome this problem, according to [33], the concept of a function must be explained visually by using set mapping diagram. This can be done by using three different representations namely, ordered pair, equation, and graph.

To have a better understanding of M1 learning experience, an interview is conducted. Here are some important results about poor teaching process and epistemological obstacles.

(i) In a class of basic calculus, the lecturer does not explain function in relation with the sets, ordered pair, Cartesian product, and relation. The following question presents M1's obstacles. Let $f(x) = x^2;\ x \in R$. Is $f(x)$ a function? The answer is negative. M1 does not really understand whether $f(x)$ is a function or not. The reason given by M1 is: "*The lecturer does not describe in details the function. She never invited the students to explore the ways to analyse the question."* According to M1, the routine question given by the lecturer is like this; "Let $f(x) = x^2 + 4$. If $x = 2$, find the value of $f(x)$*!* " Thus, it is very reasonable that M1 is facing didactical obstacles.

(ii) According to M1, the teaching and learning process do not relate to the basic concepts of function. The above fact is clear; the obstacles occur because of poor teaching process led by the lecturer in the classroom [18]. This is a warning that the lecturer must have the pedagogical knowledge, especially mathematical knowledge and skills in mathematics learning. This will help the students to develop their knowledge as well as to provide the benefits for students' mathematical achievements over time [34].

(iii) When M1 makes an example of a function, M1 does not describe the relation of the two sets A and B. Thus, it is not clear which one is the domain and which one is the co-domain. Moreover, M1 is not able to write down the function symbolically. This indicates that M1 is experiencing epistemological obstacles.

The obstacle experienced by M1 affects his/her cognitive process. On the other hand, the occurrence of obstacle shows that M1 is at the lowest level of the cognitive process. According to Bloom's taxonomy, the lowest level is the ability to remember. It is the ability to place knowledge in long-term memory that is consistent with the material presented and using relevant knowledge from long-term memory [19]. At this level, M1 stops his/her cognitive process without any development since the teachers provide information to the students without bringing the concepts of formally definition of functions.

M2's Obstacle

M2 defines a function from a set A into another set B as a relation that maps each member of A (domain) to exactly one member of B (co domain). This is fine. However, M2 has two kinds of obstacles to learning the formal definition of a function, namely, cognitive obstacle and epistemological obstacle.

The first obstacle can be viewed from conceptual understanding which relates to many topics or other representations [15]. Conceptual understanding can be regarded as a connected network of knowledge linking the related mathematical concepts [35]. In the case of a function, these concepts are set, ordered pair, Cartesian product, and relation. In this regards, cognitive obstacle occurs when one is trying to understand the definition of a function in a formal definition. And the difficulty appears when new knowledge such as sets, ordered pair, Cartesian product, and the relation is used to define function formally. This problem is related to the ability in distinguishing a relation and mapping.

Regarding epistemological obstacle, students who suffer this obstacle have no accurate manner in making an example and non-example of a function. It is then difficult for them to write the definition of a function and use mathematical symbols. This difficulty occurs because, according to [36], epistemological obstacles are in the very nature of knowledge and independent of culture, society, and learning environment.

As in the previous paragraph, an interview with M2 is conducted to have a deeper understanding about the formal definition of a function using mathematical symbols, how to link the previous knowledge and new knowledge, and non-example of function. Here are some important results.

(i) A formal definition of a function using mathematical symbols is as follows. A function is a relation f having the property that if $(a, b) \in f$ and $(a, c) \in f$, then $b = c$. This form is equivalent to this one. A function from A to B is a set f of ordered pairs in $A \times B$ where for each $a \in A$ there exist a unique $b \in B$ such that $(a, b) \in f$. In more condensed form, a function f from $A \neq \phi$ to $B \neq \phi$ written as $f: A \to B$ is $f \coloneqq \{\forall a \in A, \exists\,!\, b \in B\}$.

(ii) In addition, epistemological obstacle befalls M2. Once this obstacle occurs, M2 will not be able to connect the concepts of functions and the formal definition of a function. In this regard, [18] has remarked that students who have the difficulty in the learning process will face difficulty in carrying and linking his/her previous knowledge and new knowledge. As an example, consider two sets, *A={1, 2, 3, 4}* and *B = {2, 4, 9, 16, 25}*. Is $f(x) = x^2$ a function from *A* to *B* ? Of course not because its co domain is $\{1, 4, 9, 16\}$ which is not a subset of *B*. However, it is difficult for M2 to explain this.

(iii) Here is another non-example. Let *A = {a, b, c}* and *B = {1, 2, 3}*. Suppose *f* relates a with 1 and with 2, and simultaneously relates c with 3. It is not easy for M2 to explain that f is a non-example of function. M2 is facing the obstacle to say that $\exists x \in A$ which is mapped not exactly to one member of B. More specifically, a in A is mapped to two members of the set B. There is also a member of A, i.e. *b* which is not mapped to any member of B. On the contrary, M2 is able to explain that $x^2 + y^2 = 1$ is a circle and it is not a function since if *x* = 0, then $0^2 + y^2 = 1$ which means that there are two, is satisfying this equation, namely, *y* = 1 or *y* = – 1.

We conclude that M2 is able to represent a function in verbal, visual, and algebraic forms which shows that, according to Bloom's taxonomy, M2 is at the level of remembering and understanding.

M3's obstacle

Like M2, not only cognitive obstacles befall on M3 but also epistemological obstacles. Detailed information about this phenomenon is obtained during the interview which aims to find out how to differ the conceptual understanding that is revealed in M1, in M2, and in M3.

During the interview, we found that the way M3 students define a function is based on their understanding obtained when they were in senior high school. Thus, it is clear why it is difficult for M3 to define function symbolically. It is because their understanding has nothing to do with the concepts related to a function such as sets, ordered pair, Cartesian product, and relation. This is perhaps the reason why the conceptual understanding of these students related to the construction of example and non-example has not smoothly developed. Moreover, since M3 suffers the epistemological obstacle, M3 has no accurate manner in making an example and non-example of a function. It is then an additional difficulty to write the definition of a function and use mathematical symbols.

It can be concluded that actually, as remarked by [19], M3 should come to the level of cognitive understanding in creating or putting the ideas into a new concept of the formal definition of a function. And, according to [37], students who have reached this level will be able to understand the formal mathematical concepts. However, on the contrary, M3 is still on the level of the cognitive process of remembering. Therefore, given the many representations related to the concept of function, understanding this concept is seemingly not easy for both senior high school and college students [38]. This is the reason why many students have faced the obstacles to understanding the definition of a function. In this regards, [27] shows that students like them have difficulties in making relationships between different representations of function such as formulas, graphs, diagrams, and word descriptions. Students' obstacle in understanding those concepts rose because they found some complicacy in connecting their prior knowledge to the recent one [39].

Finally, we close this section with the question: "How can students develop their understanding of the concept of function?" To answer this question, we refer to [40] and [13]. Understanding development can be carried out through the incorporation of existing concepts and new knowledge, linking new knowledge with prior knowledge, and containing integrated knowledge structures [40]. Furthermore, to handle this problem, [13] provide two keys for learning to help students in developing conceptual understanding. These are (1) to provide opportunities for students to "try" solving the problems, and (2) to discuss conceptual relations in "explicit" manner. This is what we suggest for further action research.

6. Conclusion

It can be concluded that the students of Mathematics Education Department have two kinds of cognitive levels, these are remembering and understanding, as well as they possess three types of learning obstacles (didactical, cognitive, and epistemological). Providing students in learning process by integrating their prior knowledge with their new knowledge is a way to improve the students' cognitive level. It implies that the teachers should provide input of formal definition of a function based on the need of students at university. They should go a little beyond the students' cognitive level to make comprehensible input of formal definition of a function.

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