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Profile of the Concept Understanding of Two-Dimensional Figure Based on Pirie Kieren's Theory Reviewed from Learning Motivation in Elementary School

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Abstract: This study aims describe understanding concept of two-dimensional figure based on Pirie Kieren's theory in terms of learning motivation. This study uses a qualitative approach with a case study type of research. The results found that there were differences in the flow of students' understanding of high, medium, and low learning motivation in solving questions. When solving square questions, students with high learning motivation do effective folding back and arrive at inventing, students with motivation are doing infective folding back and not inventing, and students with low motivation do not fold back and do not get inventing. When solving rectangular problems, high-motivated students did ineffective folding back and did not arrive at inventing, moderately motivated students did not fold back and arrived at inventing, and low-motivated students did effective folding back and arrived at inventing. The similarity is that they both start from primitive knowledge.

Keywords: concept understanding, Pirie Kieren Theory, two-dimensional figure.

Abstrak: *Penelitian ini bertujuan untuk mendeskripsikan pemahaman konsep bangun datar berdasarkan teori pirie kieren ditinjau dari motivasi belajar. Penelitian ini menggunakan pendekatan kualitatif dengan jenis penelitian studi kasus untuk memaparkan secara terperinci pemahaman siswa berdasarkan teori pirie kiren. Hasil yang ditemukan terdapat perbedaan alur pemahaman siswa motivasi belajar tinggi, sedang dan rendah dalam menyelesaikan soal bangun datar. Saat menyelesaikan soal persegi siswa motivasi belajar tinggi melakukan effective folding back dan sampai pada inventising, siswa motivasi sedang melakukan inffective folding back dan tidak sampai inventising, siswa motivasi rendah tidak melakukan folding back dan tidak sampai inventising. Saat menyelesaikan soal persegi panjang siswa motivasi tinggi melakukan ineffective folding back dan tidak sampai pada inventising, siswa motivasi sedang tidak melaukukan folding back dan sampai pada inventising, siswa motivasi rendah melakukan effecting folding back dan sampai pada inventising. Persamaannya yaitu sama-sama dimulai dari primitive knowing.*

Kata kunci: *pemahaman konsep, Teori Pirie Kieren, bangun datar.*

▪ INTRODUCTION

Education is one of the efforts made to advance a nation because education can improve the quality of human resources. According to Law No. 20 of 2003 education is a conscious and planned effort to create a learning atmosphere and learning process so that students actively develop their potential so that students can have religious and spiritual strength, be able to control themselves, personality, intelligence, noble character, and skills needed by themselves, society, and the nation. Meanwhile, according to Djumali et al (2014: 1) education is an effort to prepare someone to be able to solve life problems in the present and in the future. So from the above opinion, it can be concluded that through education a person is expected to be able to develop their

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potential to be able to solve a problem through the stages of the learning process and one of the existing learning is mathematics.

However, based on the Program for International Student Assessment (PISA) (2018), it is stated that the ability of Indonesian students in mathematics, science, and reading is in low rank. For mathematics itself, Indonesia is ranked 75th out of 81 countries in the world with a score of 379. Meanwhile, in 2015 it was 385, so it looks declining. With the decline in mathematical ability in Indonesia, it can be seen that the mathematical ability of Indonesian students is low and this proves that Indonesian students are still not able to understand mathematics well. Even though the importance of understanding students' mathematics is one of the goals stated in the Regulation of the Minister of National Education of the Republic of Indonesia No. 22 of 2016 which is to understand mathematical concepts, explain the relationship between concepts, and apply concepts or algorithms in problem-solving flexibly, accurately, efficiently, and precisely.

Understanding the concept is an important basic stage in the process of learning mathematics. According to Kilpatrick et al (2001:116), conceptual understanding in mathematics is the ability of students to understand concepts, operations, and relations in mathematics. Understanding the concept needs to be applied in learning because understanding the concept will produce formulas or theorems that can be used in conditions of existing mathematical problems. Students who can understand concepts well in the learning process will find it easier to follow the learning process and be able to develop mathematical abilities so that students will have high learning achievements. However, for students who do not understand the concept, it tends to be more difficult to follow the lesson. The low ability of students in the aspect of understanding concepts is an important thing that must be followed up. Therefore, because of the importance of understanding mathematical concepts in learning, understanding concepts need to be taught to students from an early age (Herawati et al, 2010; Mulyani, 2016; Sitohang, 2018). The benefits of learning mathematics need to be directed at understanding concepts so that there are no ongoing problems for students in learning so that when students are at the junior high school level, students have no difficulty because they already have provisions since elementary school. Therefore, since elementary school learning is expected to be directed to understanding concepts so that students can interpret the material, master mathematical concepts, and are able and skilled in applying them to various situations when solving problems to realize the objectives of learning mathematics.

One of the mathematics lessons whose material is taught from elementary school to university level is geometry. In geometry material, objects are studied well, it is expected that students' verbal, visual, drawing, and logical thinking skills can grow and develop. In the mathematics of the secondary school curriculum, geometry occupies a special position because it contains many concepts (Warner, 2015). Geometry is very supportive of many other topics in mathematics. able to grow students' thinking processes (Syamsudin, 2019). Geometry learning has the aim of understanding objects directly related to facts, concepts, principles, skills, and their application in everyday life (Rinaldi, 2019). Geometry objects are objects of thought that are abstract in nature, so mastery of concepts for every teacher is very important. According to the results of research by Romansyah (2018) in elementary school students, there are 23% of high-

ability students understand the concept of a circle. 43% of moderately capable students have a sufficient understanding of the circle concept, and 34% of students have a poor understanding of the circle concept. According to Purnawanti (2013) in the results of his research on understanding the concept of triangles in elementary school students based on Van Heile's theory, it states that high-ability students have a tendency to understand geometric concepts well, and students in the medium category have sufficient conceptual understanding, low category students have a poor conceptual understanding.

From the results of research on understanding the concept of geometry in elementary school students above, it can be concluded that only students who have high-level abilities have high geometric concepts as well. Students who have a high level of ability will have no difficulty in learning or in solving mathematical problems. This is because it is influenced by various factors, one of which is interest and motivation to learn. Motivation is a psychological condition of students that encourages students to do something. Uno (2017: 23), states that learning motivation is an encouragement that comes from internal and external to students who are studying so that behavior changes occur. Students will learn effectively if they are interested in the lesson. If students are interested and interested, their learning motivation is high, so students will be easy to understand and will have no difficulty in solving math problems. So motivation has an impact and influence on students' understanding of concepts in learning. Meanwhile, based on the results of an interview with one of the teachers at SDN Pademawu Timur 5 stated that two main problems occurred, namely students' learning motivation and students mathematical abilities which were classified as low because students considered mathematics to be a difficult subject, causing students not to like mathematics and at the time of learning. did not pay much attention so they did not understand the material being explained.

At the time paying attention to and understanding the concept of mathematical material students experience a complex and meaningful inner thinking process so that student understanding develops. The growth or development of this understanding is explained by Susan Pirie and Thomas Kieren that mathematical understanding is an understanding that develops through multilevel levels, but is not linear. This is an important theory in mathematics education because it provides deep insight into the meaning of understanding something. (Lester, 2005) This Pirie Kieren's theory consists of 8 layers and each layer includes all subsequent layers and development moves outward (Martin, 2008). It can be interpreted that gaining understanding can be obtained from anywhere according to their thinking. Moreover, Pirie (1989) claims, and illustrates from the extensively recorded interactions of students doing mathematics, that understanding mathematics is a complex phenomenon for students who do it. This theory can find out how students think about things that go beyond what is seen. One must consider the individual experiences, perceptions, and interactions that students have in their environment when analyzing the process of understanding mathematics from a constructivist perspective (Pirie, 1992). Many researchers have looked at learning from this perspective and provided different theories to characterize mathematical understanding (Davis, 1984; Sfard, 1991; Sierpinska, 1994; Skemp, 1978). In addition, research based on Pirie and Kieren's theories is not as much as other understanding theories. There are previous studies that are relevant and have been carried out by other researchers about describing understanding concepts based on Pirie

and Kieren's theories. Hasanah (2019) with his research aims to describe students' understanding of geometric concepts (build space) based on Pirie and Kieren's theories in terms of students who have high, medium, and low concept understanding categories. The results of the study stated that students who have a high level of understanding of geometric concepts meet eight layers of Pirie and Kieren theories, while students who have a moderate understanding of geometric concepts only meet 7 layers, and students who have a low understanding of geometric concepts only reach one level. Suindayanti et al (2020) with their research aim to describe the layers of understanding of junior high school students who are highly capable of solving spatial problems based on Pirie and Kieren's theories. The results of his research stated that students who had high abilities were able to meet all the indicators of the Pirie and Kieren theory layers. Asih et al (2020) with the aim of their research, namely to describe the layer profile of understanding the concept of sequences and series based on the Pirie Kieren theory in junior high school students. The result of the research is to describe each layer of students' understanding in understanding the concepts of lines and series.

However, the research that wants to be carried out in this study is to describe students' conceptual understanding based on the Pirie Kirien theory in terms of learning motivation in elementary students. two-dimension figures were chosen by researchers because they are the basis for learning geometry which must be understood before learning geometric shapes. In addition, research on understanding the concept of squares, rectangles, and triangles in elementary school students based on Pirie Kieren's theory has not been carried out by other researchers. Relevant previous research is more on the material of building space. Therefore, based on the above background, the researcher is interested in conducting a study entitled: Profile of students' understanding of the flat wake concept based on the Pirie Kieren theory in terms of learning motivation in elementary school.

▪ METHOD

The research uses a qualitative approach. Sugiyono (2015:15) qualitative research is based on the philosophy of postpositivism, which is used in the condition of natural objects, and the researcher is the key instrument, and the sampling of data sources is carried out by purposive sampling, namely sampling by providing special characteristics that are suitable for the purpose. study. This research was carried out at the Pademawu Timur 5 Public Elementary School located in East Pademawu Village, Pademawu District, Pamekasan Regency.

Paticipants

In this study, 21 fourth-grade students of SDN Pademawu Timur 5 will be given a learning motivation questionnaire and classified into low, medium, and high levels of learning motivation. Then 3 subjects will be taken in this study, each of which consists of 1 student who has high, medium, and low learning motivation. Subjects will be taken using the purposive sampling method. Purposive sampling is a sampling technique with certain considerations or also called purposive sampling (Ikhsan 2008). The criteria set are students must be communicative, and willing to be given an understanding test and interview.

Research Design and Procedures

The research design used in this research is a case study. The type of research case study according to Yin (2013: 18) is an empirical inquiry that investigates phenomena in real-life contexts, when the boundaries between phenomena and contexts are not visible and where multiple sources of evidence are utilized. Case studies are used in this study to be able to explore in depth the understanding of students' concepts on flat-shaped material based on Pirie Kieren's theory in terms of learning motivation.

The first step in this research is to determine the research population, namely 21 fourth-grade students, the next step is to provide a learning motivation questionnaire to classify students' learning motivation levels into high, medium, and low learning motivation, then take a sample of 3 students, each consisting of 3 students. from 1 student with high, medium, and low motivation with the criteria of being communicative, willing to be given a test and being interviewed, the last stage, namely the 3 students were given a flat wake comprehension test and interviewed. The interview method used in this study is a semi-structured interview method. Semi-structured interviews refer to interview guidelines in the form of questions that will be asked in the hope of obtaining adequate data about understanding concepts based on Pirie Kieren's theory in terms of learning motivation. If at the time of the interview there was still information that was felt to be lacking, the researcher asked questions outside the interview guidelines that had been prepared. And the interview activities were recorded through audio-visual, this was done to avoid the loss or missed information to be obtained. Data collection in this study was carried out in at least two stages. It aims to obtain valid data. To check the validity of the conceptual understanding test and interviews in this study, time triangulation was carried out. The period of this research is 30 May-12 June 2022

Instrument

The instruments used in this study were a learning motivation questionnaire, a test for understanding the concept of flat shapes, and interviews. The learning motivation questionnaire instrument in this study is a closed questionnaire type because it only needs to put a checkmark on one of the answers that are considered appropriate for him. The learning motivation questionnaire instrument consists of 20 statements that are adjusted to the learning motivation indicators from Uno (2017). The grouping of students' learning motivation uses descriptive statistics on average based on the value given and the range of each category is determined using statistical equations that are adjusted to the data (Astupura et al, 2016). The number of aspects observed is 20, the maximum score is 100 and the minimum score is 20 with an interval of 4 so the grouping of learning motivation consists of a score range of 20-40 which is included in very low learning motivation, a score range of 41-60 is included in low learning motivation. a score of 61-80 for moderate learning motivation, and a score range of 81-100 including high learning motivation performance. The concept understanding test instrument contains 2 questions about squares and rectangles. The interview guide instrument contains a collection of customized questions that will be asked to students to determine student understanding in depth. Before using this research instrument, the feasibility of this research was tested first, which was validated by one of the lecturers of mathematics education at the University of Madura and a fourth-grade teacher at the Pademawu Timur State Elementary School V.

Data Analysis

Data analysis in this study refers to Miles & Hiberman (1994), which includes data reduction (data reduction), data presentation (data display), and conclusion drawing (conclusion drawing/verification. Data reduction in this study is correcting and analyzing the student's understanding of the flat shape concept test, the results of the student's work were then transformed into notes for further questioning students, and the results of interviews with each research subject were simplified into a good language structure and analyzed. and presenting the results of interviews and data exposure results, then concluding the form of a description of students' understanding of the concept of flat shapes based on Pirie Kieren's theory in terms of learning motivation.

▪ RESULT AND DISSCUSSION

Understanding the concept of two-dimensional figure is the ability of students to understand ideas in two-dimensional figure in the form of understanding, characteristics, nature, and properties. Two-dimensional figure are shapes that have a flat surface and have length, width, area, and circumference. Pirie Kieren's theory is a theory of understanding that states that students' understanding of mathematics is a multilevel understanding but not linear. The indicators of understanding the concept and theory of Pirie Kieren have in common (Hasanah, 2019) as follows:

Table 2. Indicators of concept understanding based on pirie kieren's theory

No	Indicators of Concept Understanding	Pirie Kieren's Theory
1	Able to restate a concept	*Primitive knowing* (students have a simple knowledge beginning/knowledge)
2.	Able to classify objects by certain characteristics (by concepts)	*Image making* (students were able to make an idea of the initial knowledge he had)
3	Able to classify objects by certain characteristics (by concepts)	*image having* (students were able to make an idea of the initial knowledge he had)
4	Being able to provide examples and non-examples	*Property noticing* (students have a view and have their own understanding)
5.	Capable of presenting concepts in various forms of mathematical representation	*Formalizing* (the student is able to develop aspects he knows to form relevant qualities.)
6.	The student is able to develop the necessary conditions or requirements of enough a concept	*Observing* (students are able to form a concept of an existing trait)
7.	Students are able to use, utilize, and select specific procedures or operations	*Structuring* (students were able to link one theorem with another and were able to give a logical reason)
8.	Students are able to apply concepts to solve problems	*Inventising* (students are able to have a complete structural understanding)

Source: Hasanah (2019)

Alternating from tests and interviews that researchers conducted at the country's elementary school at Pademawu east 5, an understanding of students' concepts on a flat build based on Pirie Kieren's theory reviewed from learning motivation is as follows:

Profile Understanding Two-Figure Dimension Concept Of Highly Motivated Students Based On Pirie Kieren's Theory

Based on the results of Test 1 and Test 2, understanding the concept of a square based on the Pirie Kieren theory on SP subjects who have high learning motivation, namely SP subjects understand the meaning of the problem by knowing the area of the square in the problem, the definition of area and the definition of the perimeter, and this shows that SP subject can restate the concept of a square (Primitive knowing), besides SP subject makes a square shape as an example of a garden, this shows that SP subject can know the shape of a flat square shape (Image making), then SP subject looks for the length of the side of the square through the area of the square. What is known in the problem, shows that the subject of SP has developed the necessary and sufficient conditions in calculating a concept where the area of a square is a sufficient condition and side length is a necessary condition (Observing). The SP subject calculates the side length by trying to multiply the same number, this shows that the SP subject can classify objects according to certain properties. Furthermore, the subject of SP relates the length of the side found to the perimeter of the square and the subject of SP can give reasons for the relationship by stating that the circumference is the sum of its sides and hereby shows that the subject of SP can use, utilize and choose certain procedures or operations (Structuring). Finally, the subject of SP concludes the results of the answers he finds and this proves that the subject of SP can apply concepts or problem-solving algorithms (inventing). At the time of understanding the square question, the subject of SP met 6 indicators of conceptual understanding. This is following Hasanah (2019) in her research entitled The profile of understanding geometric concepts based on the Pirie Kieren theory stating that if it meets 6 of the 7 indicators of concept understanding, it can be concluded that students have sufficient understanding abilities.

Based on the results of Test 1 and Test 2 understanding of the concept of a rectangle, SP subjects who have high learning motivation, namely SP subjects understand the problem by knowing the area of the rectangle, the area of the rectangle, and this shows that the SP subject is able to restate the concept (Primitive knowing), make rectangular image as an example of the existing problems, this shows that the subject of SP is able to know the shape of a rectangular image (Image making), After making a picture of the subject SP understands that the area of the yard is the area of a flat rectangular field, this shows that the subject is able to give example (Property noticing), then the subject of SP calculates the area of the yard using the concept of a rectangular area, this shows that the subject of SP is able to use, utilize and choose procedures in solving the problem (Structuring), then the last one is because the subject of SP wants to make sure that the answers answered right, then what the SP subject did huh it calculates the circumference with the known side length and then matches it with the known circumference in the problem, this shows the SP subject classifying the circumference based on the right and wrong circumference (Image making). When described in nested circles, the following is a layer of understanding of the concept of SP subjects who have high learning motivation based on the Pirie Kieren theory:

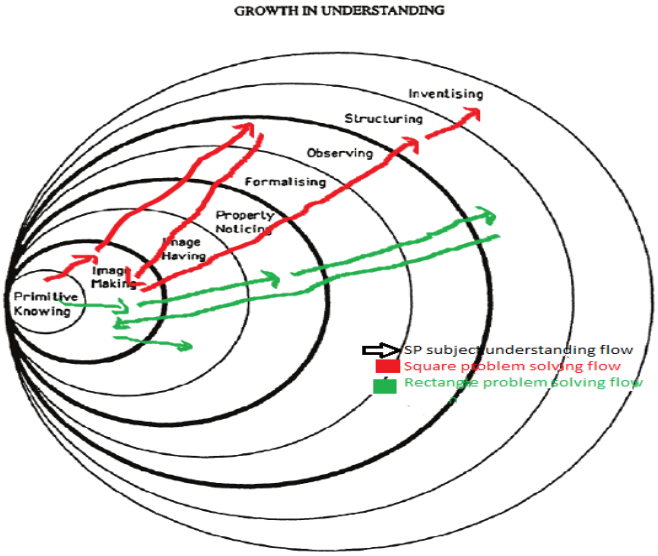

Figure 1. Layers of understanding the concept of Square and rectangle by Subject SP

From the development of the SP subject's understanding of the two square and rectangular materials above, if the image of the understanding layer is considered, the SP subject returns to the deepest layer when solving problems related to squares and rectangles. The return of the understanding of the SP subject to the inner layer is following Pirie & Kieren (1994) who states that students have time to return to the deepest layer when facing a problem and this is called folding back. Susiswo (2014) states that the return of students' conceptual understanding to the deepest layer can occur when students are faced with a problem but cannot quickly solve it. In the layer flow of understanding the concept of the SP subject to the square, the SP subject returns to the inner layer of image making when solving the square problem and the SP subject returns to the deepest layer of property noticing and image making when students complete the rectangular question. Slaten (2010) explains that there are effective folding back and ineffective folding back. Effective folding back when someone can use the expansion of understanding gained to solve existing problems. while ineffective folding back when one is unable to use the understanding one has acquired. So it can be concluded that the SP subject at the time of completing the square experienced 1 time effective folding and 1 time effective folding back when solving rectangular problems.

Profile Understanding Two-Figure Dimension Concept Of Moderated Motivated Students Based On Pirie Kieren's Theory

Based on the results of test 1 and test 2 understanding the concept of a square FL subject has moderate learning motivation, namely understanding the shape of a square, the formula for the area of a square, and the perimeter of a square. square is the side times the side and the perimeter of the square is 4 times the side. This proves that the FL subject is able to restate a concept, namely the concept of a square (Primitive knowing), the FL subject does the steps by connecting the side lengths to the area of the square and the perimeter of the square by first finding the side length through the area of the square and this shows that the subject FL is able to develop the necessary requirements, namely

side length, and sufficient concept requirements, namely the perimeter of a square (Observing), after finding the side length value of the subject, FL substitutes the side length value into the circumference formula and this shows the ability of the FL subject to apply the concept (Structuring), the subject of FL confused because there is still an s value that appears then the FL subject makes a square image to make a picture that can make it easier to find the sides and from this it shows that the subject is able to identify the square shape (image making), the FL subject returns to the primitive knowing stage to find out if the area is wide. square sides times s the content and this shows the subject of FL again restating a square concept. The results of research conducted by Nillas (2010) and Wilson and Stein (2007) support this finding. They suggest that students may need to work on a deep level of mathematical understanding to build relationships among various representations of mathematical concepts. From primitive knowing, the subject gets an idea, namely finding the side of a square by trying to find and multiply the number which, if multiplied, is following the known area of the problem, this shows that the FL subject can classify objects according to their nature (Image making), FL subjects have not been able to find answers with the right because the subject of FL only memorizes formulas and does not understand the nature of squares having equal sides. This is in line with the statement of Jaeng (2015) in his research entitled "Profile of understanding the concept of elementary school students in solving the area and circumference of a circle" stating that if you only remember formulas, then the ability you have is only in mastering facts. however, when the subject of FL has not found the right answer, the subject of FL still has a picture by trying to multiply it to find the area of the square (Image having). Gulkilik et al (2015) stated that the return of the deepest layer directly affects the growth of students' understanding. This is in line with the FL subject which returns to the primitive knowing layer but can reach the image having layer.

Based on the results of test 1 and test 2 understanding the concept of a rectangle on FL subjects who have moderate learning motivation, namely knowing the area and circumference of a rectangle, area of a rectangle, length of a rectangle besides that, FL subjects understand the characteristics of a rectangle that has 2 opposite sides of the same length and this shows that the FL subject is able to restate the concept of a rectangle (Primitive knowing), the FL subject calculates the width of the rectangle through the perimeter of the rectangle which is known directly without using a formula but using the students' understanding of the circumference of a flat figure, namely circumference is to add up the sides of this shows that the subject is able to develop the necessary conditions, namely the perimeter of the rectangle and the length of the rectangle and the sufficient condition, namely the width of the rectangle (Observing), The ability of students to make connections between their understanding of the perimeter of a square to calculate the perimeter of a rectangle. In terms of terms and expressed in symbolic terms, it shows that students have relational understanding (Keene, 2011) then the FL subject applies the width of the rectangle obtained to determine the area of the rectangle (Structuring). After the area of the rectangle is known, the subject of FL concludes that calculating the area of a rectangle is the same as calculating the area of a rectangle (inventing).

When described in nested circles, the following is a layer of understanding of the concept of FL subjects who have moderate learning motivation based on Pirie Kieren's t theory:

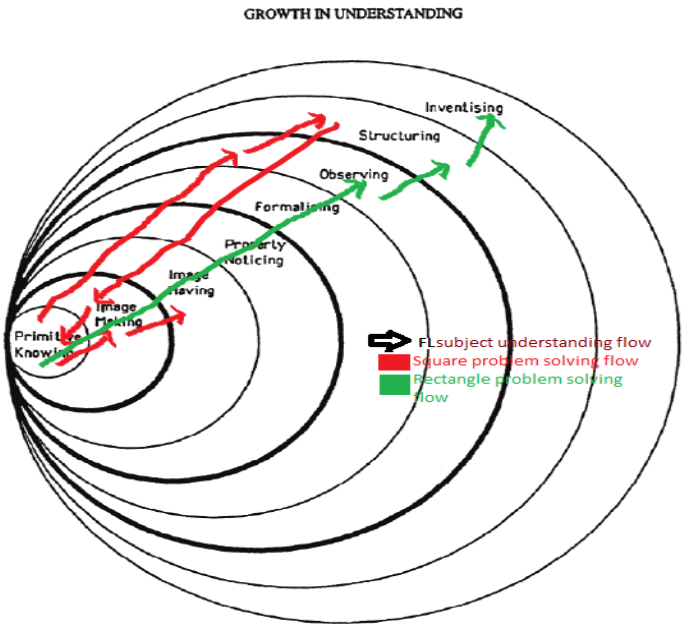

Figure 2. Layers of understanding the concept of Square and rectangle by Subject FL

From the flow of understanding of the FL subject above, it can be seen that when solving the square problem, the FL subject does the following inffective folding back 2 times and effective folding back 1 time. Meanwhile, when solving rectangular problems, FL subjects did not folding back

Profile Understanding Two-Figure Dimension Concept Of Low Motivated Students Based On Pirie Kieren's Theory

Based on the Results of Work on Tests 1 and 2 Understanding the concept of a square on NP subjects who have low learning motivation, namely understanding the meaning of the question well, namely stating what is understood about the area and perimeter of the square on the question, this shows the subject can restate a concept (primitive knowing), the subject brings knowledge that a square has 4 equal sides then the subject of NP describes a square. According to Oscakir (2019), students' ability to express the nature of a square is a stage where students can recognize shapes and define and describe squares. This shows that the subject can distinguish which objects are square and which are not (image making), the subject defines the area of a square in another way, namely by describing a square with small squares that fill its area, this shows the subject can state and understand the definition of a square area without using formulas but by using pictures or presenting concepts in various mathematical representations (Formalizing). Wahyu et al (2019) stated that using a square unit as a unit area is a common way to help students build their understanding of the concept of a topic in mathematics. After that, the subject looks for the perimeter of the square by first

calculating the side lengths, this shows that finding the length of the sides of the square is a necessary condition to know the circumference (Observing), the subject looks for the side length through a known area, this shows the subject's ability to solve problems by using and utilizing and selecting certain procedures (Structuring).

The NP subject stated that the subject understood the meaning of the problem, namely the perimeter of the rectangle, the length of the rectangle and the area of the rectangle, this shows that the subject of NP is able to restate a concept (primitive knowing), then the subject draws a rectangle as a description of the problem in the problem, this shows that the NP subject shows his knowledge in identifying rectangular shapes (Image making), the NP subject calculates the circumference of the circumference without using a formula but the NP subject already understands the definition of circumference by adding up all the sides, this shows the NP subject is able to develop the necessary conditions and the requirement is sufficient for a concept (Observing), NP subjects draw a rectangle and write down the length and width and draw a small square on the side of the field, this shows that the NP subject is able to present the concept of area not only using formulas (Formalising), lastly the subject NP calculates the area of a square i length by multiplying the known length of the problem and the width that has been found and concludes that the area of the given problem is the same as calculating the area of a rectangle (inventing). Students need folding back when they cannot immediately solve problems with the help of their current understanding and this movement gives them to expand their mathematical understanding so that students can move to the outer layer (Martin, 2016; Lawan, 2011)

When described in nested circles, the following is a layer of understanding of the concept of NP subjects who have low learning motivation based on pirie kieren theory:

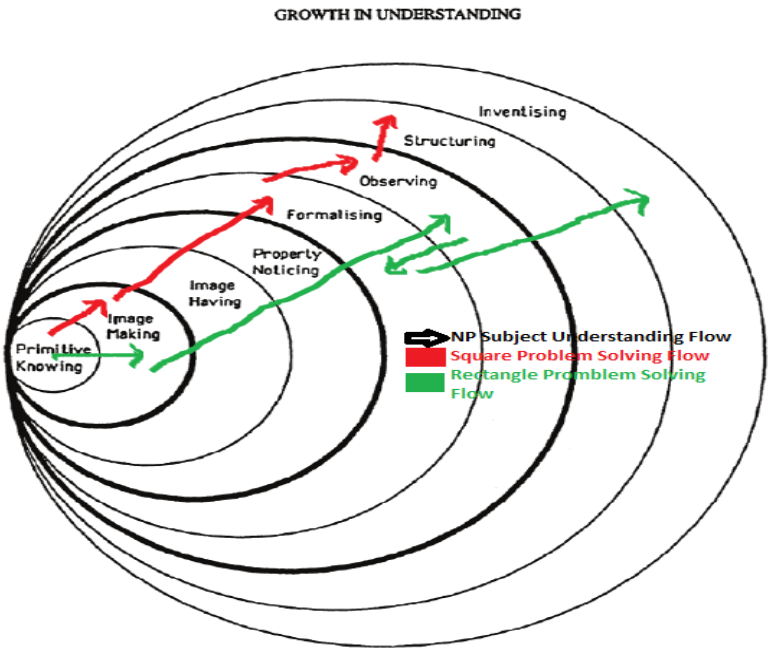

Figure 3. Layers of understanding the concept of Square and rectangle by Subject NP

▪ CONCLUSION

Based on the results of data analysis and discussion of the three research subjects, a description of the profile of understanding the concept of flat shapes based on the pirie kieren theory is obtained, namely SP subjects who have high learning motivation through layers of understanding. primitive knowing, to observing, do effective folding back from observing to image making, to structuring and end up in inventising. While on the subject of SP subject rectangles through layers of understanding from primitive knowing to property noticing to structuring, do effective folding back from structuring to image making then end up in image having. FL subjects who have moderate learning motivation, namely through layers of understanding from primitive knowing to observing then structuring do ineffective folding back from structuring to image making then do effective folding back from primitive knowing to image making, continued to image having. While in the subject of FL rectangular material through layers of understanding from primitive knowing, to structuring then inventising. NP subjects who have low learning motivation, namely through layers of understanding from primitive knowing to image making then to formalising, observing and structuring. While the profile of understanding the concept of a rectangular NP subject through the understanding layer from primitive knowing, to observing, then do effective folding back from observing to formalising then to inventising. The equations of the 3 subjects in solving square and rectangular problems are both starting from the primitive knowing stage. Similarly, many studies reveal that mathematical understanding occurs based on prior mathematical concepts and knowledge and primitive knowing is very important for understanding (Grinevitch, 2004; Hollebrands, 2003; Pirie, 1994).

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